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Contents • MHR vii
How is *Mathematics 8: Making Connections* set up?

Each chapter starts off with a Chapter Problem that connects math and your world. You will be able to solve the problem using the math skills that you learn in the chapter.

You are asked to answer questions related to the problem throughout the chapter.

The Get Ready pages provide a brief review of skills from previous grades that are important for success with this chapter.

The numbered sections often start with a photo to connect the topic to a real setting. The purpose of this introduction is to help you make connections between the math in the section and the real world, or to make connections to previous knowledge.

The Chapter Problem Wrap-Up is at the end of the chapter, on the second Practice Test page.
A three-part lesson follows.

**Discover the Math**

Is there a constant relationship among measures of sides and angles?

1. Choose five circular objects. Measure around the outside of each. What measurement is this?

**Key Ideas**

- The measures of the internal angles in a triangle add to 180°.
- You can use the sum of the internal angles to find unknown measures in triangles.

**Check Your Understanding**

**Practise**

For help with questions 4 and 5, refer to Example 1.

4. What is the unknown angle measure in each triangle?
   - a)  
   
5. a) Copy each equation and use the symbols +, −, ×, ÷, and ( ) to make it true.
   - 5 + 9 − 7 = 3
   - 3 × 5 − 4 = 8
   - 20 − 4 + 6 = 4
   b) Can you make any of the equations true in more than one way? Explain.
   c) What strategies did you try to answer these questions? Which worked best?

15. a) Copy each equation and use the symbols +, −, ×, ÷, and ( ) to make it true.
   b) Can you make any of the equations true in more than one way? Explain.
   c) What strategies did you try to answer these questions? Which worked best?

**9.4 Use Databases to Solve Problems**

**Explore the Pythagorean Relationship Using The Geometer’s Sketchpad®**

The first part helps you find answers to the key question.

- An activity is designed to help you build your own understanding of the new concept and lead toward answers to the key question.

- Examples and Solutions demonstrate how to use the concept.

- A summary of the main new concepts is given in the Key Ideas box.

- Questions in the Communicate the Ideas section let you talk or write about the concepts and assess whether you understand the ideas.

- Practise: these are straightforward questions to check your knowledge and understanding of what you have learned.

- Apply: in these questions, you need to apply what you have learned to solve problems.

- Extend: these questions may be a little more challenging and may make connections to other lessons.

The last question in each set of questions is designed to assess your level of success with the section. Everyone should be able to respond to at least some part of each question.

Numbered sections that have a green tab are based on the use of technology such as scientific calculators, spreadsheets, or The Geometer’s Sketchpad®.

Some numbered sections are followed by a Use Technology feature. This means some or part of the preceding section may be done using the technology shown.
How does *Mathematics 8: Making Connections* help you learn?

**Understanding Vocabulary**

Key words are listed on the Chapter Opener. Perhaps you already know the meaning of some of them. Great! If not, watch for these terms highlighted the first time they are used in the chapter. The meaning is given close by in the margin.

[Literacy Connections provide tips to help you read and interpret items in math. These tips will help you in other subjects as well.]

**Understanding Concepts**

The Discover the Math activity is designed to help you construct your own understanding of new concepts. The key question tells you what the activity is about. Short steps, with illustrations, lead you to be able to make some conclusions in the last step, the Reflect question.

You have explored what is called the Pythagorean relationship. The sum of the areas of the squares on the two shorter sides, or legs, of a right triangle is equal to the area of the square on the hypotenuse.

![Diagram of a right triangle with labels for legs and hypotenuse.]

**Literacy Connections**

Writing Answers

Always include proper units in the final answer to any measurement problem.

**Discover the Math**

How can you multiply integers using patterns?

**Part 1: Multiply Opposite Integers**

1. Copy and complete the multiplication statements to continue the pattern.
   - $4 \times 3 = 12$
   - $4 \times 2 = \text{ }$
   - $4 \times 1 = \text{ }$
   - $4 \times 0 = \text{ }$
   - $4 \times (-1) = \text{ }$
   - $4 \times (-2) = \text{ }$

2. Describe the pattern. Explain how you used it to complete the last two multiplications.

3. Reflect
   - a) State a rule for multiplying a positive number by a negative number.
   - b) Would your rule hold for a negative number times a positive number? Explain.
The Examples and their worked Solutions include several tools to help you understand the work.

- Notes in a thought or speech bubble help you think through the steps.
- Sometimes different methods of solving the same problem are shown. One way may make more sense to you than the other.
- Problem Solving Strategies are pointed out.
- Calculator key press sequences are shown where appropriate.

The exercises begin with Communicate the Ideas. These two or three short questions focus your thinking on the Key Ideas you learned in the section. By discussing these questions in a group, or doing the action called for, you can see whether you understand the main points and are ready to start the exercises.

The first few questions in Check Your Understanding can often be done by following one of the worked Examples.

**Example 2: Apply Rates to Commission**

Refer to Example 1. Devon hopes to earn $1000 by the end of the summer. What total value must he sell?

**Solution**

**Method 1: Use Equivalent Ratios**

40% commission means that he earns $40 for every $100 of sales.

\[
\frac{40}{100} = \frac{x}{1000}
\]

To earn $1000 in commission, Devon must sell a total of $2500.

**Method 2: Use Systematic Trial**

40% is less than half. To earn a $1000 commission, Devon's total sales must be a little more than double $1000. Start by trying sales of $2200.

\[
\begin{array}{c|c|c}
\text{Total Sales} & \text{Commission (40% of Total Sales)} & \text{Sales} \\
$2200 & $880 & $2200 \\
$2300 & $920 & $2300 \\
$2500 & $1000 & $2500 \\
\end{array}
\]

To earn $1000 in commission, Devon must sell a total of $2500.

**The Pythagorean relationship connects the three sides of any right triangle.**

**Communicate the Ideas**

1. a) Describe the steps you would use to find the length \( x \).
   b) Estimate the value of \( x \). Between which two whole numbers is it?

2. To find the length \( b \), Crystal wrote

\[ b^2 = 10^2 + 8^2 \]

Is her method correct? If not, explain what she did wrong.

3. Darian wants to find the height, \( h \), of this triangle. His method is to enter \( \sqrt{81 - 49} \) into his calculator. Will his answer be correct? Explain why or why not.

**Check Your Understanding**

5. Find the length of the missing side in each triangle.

4. Find the length of the missing side in each triangle.

**Practise**

For help with questions 4 to 7, refer to Examples 1 and 2.
What else will you find in *Mathematics 8: Making Connections*?

Two special sections at the beginning of the book will help you to be successful with the grade 8 course.

**Problem Solving**

This is an overview of the four steps you can use to approach solving problems. Samples of 12 problem solving strategies are shown. You can refer back to this section if you need help choosing a strategy to solve a problem. You are also encouraged to use your own strategies.

---

**Problem 1**

Dina's family owns and operates a small restaurant. They have many small square tables and folding chairs. What is the greatest number of people that can be seated when 10 tables are put together?

**Problem 2**

Amy's mother bought a basket of strawberries. Ben came in and ate half of them. Steve came home next and ate half of the remaining strawberries. Dora returned and ate half the number that remained. Amy came home last. She ate half of the remaining strawberries and left two whole strawberries for her mother. How many strawberries were originally in the basket?

---

**Get Ready for Grade 8**

These six pages present a brief review of basic concepts from earlier grades and ways of thinking about the concepts.

---

**Did You Know?**

These are interesting facts related to math topics you are learning.

**Making Connections**

These activities link the current topic to careers, games, or to another subject.

**Internet Connect**

You can find extra information related to some questions on the Internet. Log on to [www.mcgrawhill.ca/links/math8](http://www.mcgrawhill.ca/links/math8) and you will be able to link to recommended Web sites.
Each chapter ends with a **Chapter Review** and a **Practice Test**. The chapter review is organized by section number so you can look back if you need help with a question. The test includes the different types of questions that you will find on provincial tests: multiple choice, short answer, and extended response.

**Task**

These projects follow each pair of chapters. To provide a solution, you may need to combine skills from multiple chapters and your own creativity.

---

**Design a Runway**

To raise money for a spring trip, your school will host a fashion show in the gym. Design a sloping runway so that the models can walk through the audience and up on to the stage.

Your research suggests that:
- the stage should be 1 m high
- the area of the stage should be 36 m²
- the runway must be wide enough for two models to pass each other
- the runway must be centred on the stage
- the runway must be painted
- the surface of the runway must have slip-proof matting
- the runway needs to be packed with material to reduce vibrations

1. Draw labelled diagrams of your design.

---

**Reviews** of the previous four chapters can be found following Chapters 4, 8, and 12.

**Answers**

Answers are provided to the odd-numbered Practise, Apply, and Extend questions, as well as, Reviews and Practice Tests. Sample answers are given for questions that have a variety of possible answers or that involve communication. If you need help, read the sample and then try to give an alternative response.

Answers are omitted for the Try This and the Chapter Problem questions because teachers may use these questions to assess your progress.

**Glossary**

Refer to the illustrated Glossary at the back of the text if you need to check the exact meaning of mathematical terms.
How can you solve problems like the four below? Compare your ideas with the strategies that are shown on the following pages.

Problem 1
Dina’s family owns and operates a small restaurant. They have many small square tables and folding chairs. What is the greatest number of people that can be seated when 10 tables are put together?

Problem 2
Amy’s mother bought a basket of strawberries. Ben came in and ate half of them. Steve came home next and ate half of the remaining strawberries. Dora returned and ate half the number that remained. Amy came home last. She ate half of the remaining strawberries and left two whole strawberries for her mother. How many strawberries were originally in the basket?

Problem 3
A road crew is repainting the solid yellow line down the centre of the road from Owen Sound to Tobermory. 1 L of the paint covers 4 m². How many litres of paint does the crew need?

Problem 4
Raj and his friend Matt live in a neighbourhood where the streets form a regular grid pattern. How many different routes are there from Raj’s house to Matt’s? Assume that Raj does not retrace his steps and always takes the shortest route.
People solve mathematical problems at home, at work, and at play. There are many different ways to solve problems. In *Mathematics 8: Making Connections*, you are encouraged to try different methods and to use your own ideas. Your method may be different but it may also work.

**A Problem Solving Model**

Where do you begin with problem solving? We suggest the following four-step process.

**Read the problem.**
- Think about the problem. Express it in your own words.
- What information do you have?
- What further information do you need?
- What is the problem asking you to do?

**Select a strategy for solving the problem.** You may sometimes need more than one strategy.
- Consider other problems you have solved successfully. Is this problem like one of them? Can you use a similar strategy? Strategies that you might use include
  - Make a picture or diagram
  - Make an organized list
  - Look for a pattern
  - Make a model
  - Work backward
  - Make a table or chart
  - Consider whether any of the following might help. Plan how to use them.
    - tools such as a ruler or a calculator
    - materials such as graph paper or a number line

**Solve the problem by carrying out your plan.**
- Use mental math to estimate a possible answer.
- Do the calculations and record your steps.
- Explain and justify your thinking.
- Revise your plan if it does not work out.

**Examine your answer.** Does it make sense?
- Is your answer close to your estimate?
- Does your answer fit the facts given in the problem?
- Is the answer reasonable? If not, make a new plan. Try a different strategy.
- Consider solving the problem a different way. Do you get the same answer?
- Compare your method with that of other students.
Here are some different ways to solve the four problems on page xvi. Often you need to use more than one strategy to solve a problem. Your ideas on how to solve the problems might be different from any of these.

To see other examples of how to use these strategies, refer to the page references. These show where the strategy is used in other sections of *Mathematics 8: Making Connections*.

### Problem 1

Dina's family owns and operates a small restaurant. They have many small square tables and folding chairs. What is the greatest number of people that can be seated when 10 tables are put together?

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<td>Look for a pattern</td>
<td><img src="image" alt="Pattern" /></td>
<td>pages 187, 199, 215</td>
</tr>
</tbody>
</table>

When 10 tables are put together in a line, \(4 + 2 \times 9\) people can be seated. This is 22 people.

When 3 tables are put together in an L-shape, 8 people can be seated. This is the same as when the tables are in a line.

When 4 tables are put together to form a square, 8 people can be seated. This is less than when the tables are in a line.

**Conclusion:** The greatest number of people that can be seated when 10 tables are put together is 22 people.
Problem 2

Amy’s mother bought a basket of strawberries. Ben came in and ate half of them. Steve came home next and ate half of the remaining strawberries. Dora returned and ate half the number that remained. Amy came home last. She ate half of the remaining strawberries and left two whole strawberries for her mother. How many strawberries were originally in the basket?

### Strategy

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
<th>Other Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work backward</td>
<td>Two strawberries</td>
<td>page 200</td>
</tr>
<tr>
<td></td>
<td>were left for Amy’s mother. The last person, Amy, must have eaten two. Half of 4 is 2. That’s right. I will work backward from there.</td>
<td>pages 167, 1688, 282,401</td>
</tr>
<tr>
<td>Make a table or chart</td>
<td><img src="image" alt="Table" /></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Person</th>
<th>Number Eaten</th>
<th>Remaining</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amy</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Dora</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Steve</td>
<td>8</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Ben</td>
<td>16</td>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>

**Look Back** Check that a basket containing 32 works.

Ben: half of 32 = 16
Steve: half of 16 = 8
Dora: half of 8 = 4
Amy: half of 4 = 2
Mother: 2
16 + 8 + 4 + 2 + 2 = 32 It checks.
There were 32 strawberries in the basket originally.

### Act it out

- Mother sees
- Amy sees
- Dora sees
- Steve sees

I can act this out, using red counters to represent the strawberries.

In the full basket, there were 32 strawberries.

### Work backward

- Ben: half of 40 = 20
- Steve: half of 20 = 10
- Dora: half of 10 = 5

**Look Back** I need to change my starting number, because Amy would be eating 2 1/2 strawberries and leaving the same number for her mother. The problem says the mother got 2 whole berries.

Since 2 is less than 2 1/2, I will start with a smaller number.

When I repeat the steps with 32, I find it works perfectly.
There were 32 strawberries in the basket originally.
**Problem 3**

A road crew is repainting the solid yellow line down the centre of the road from Owen Sound to Tobermory. 1 L of the paint covers 4 m². How many litres of paint does the crew need?

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
<th>Other Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make an assumption</td>
<td>I’ll assume that just one coat of paint is used.</td>
<td>page 233</td>
</tr>
<tr>
<td>Find needed information</td>
<td>I need to find the distance from Owen Sound to Tobermory. I can use an Ontario road map. I can find the width of the centre yellow line by measuring one near my school.</td>
<td>pages 193, 233</td>
</tr>
<tr>
<td>Choose a formula</td>
<td>The yellow line is approximately a long thin rectangle. I need to find its area. I am told that 1 L of paint covers 4 m², so I need to work in metres. 113 km = 113 × 1000 m 10 cm = 0.10 m</td>
<td>pages 38, 69, 171, 254, 442</td>
</tr>
</tbody>
</table>

It is 113 km from Owen Sound to Tobermory.

The yellow line is 10 cm wide.

\[ A = l \times w \]
\[ A = 113 \times 1000 \times 0.10 \]
\[ A = 11300 \]

The area of the centre line is 11300 m².

Number of litres = \[ \frac{11300}{4} \]

= 2825

To paint the centre line from Owen Sound to Tobermory, the crew needs approximately 2825 L of paint.

**Look Back**

Each kilometre is 1000 m.

1000 m × 0.10 m = 100 m²

For each kilometre, 100 ÷ 4 or 25 L of paint are needed.

So, for 100 km, 2500 L of paint are needed. This shows that the calculated answer is reasonable.
Problem 4  Raj and his friend Matt live in a neighbourhood where the streets form a regular grid pattern. How many different routes are there from Raj’s house to Matt’s? Assume that Raj does not retrace his steps and always takes the shortest route.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
<th>Other Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve a simpler problem</td>
<td>What if they lived one block apart?</td>
<td>pages 200, 234</td>
</tr>
<tr>
<td>Look for a pattern</td>
<td>There are 2 possible routes in this case, each 2 blocks long.</td>
<td>pages 361, 401</td>
</tr>
<tr>
<td></td>
<td>What if they lived two blocks apart?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>There are 6 possible routes in this case, each 4 blocks long.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>What if they lived three blocks apart?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>There are 20 possible routes in this case, each 6 blocks long.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The last step: They actually live four blocks apart.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>There are 70 possible routes in this case, each 8 blocks long.</td>
<td></td>
</tr>
</tbody>
</table>

Amazingly there are 70 possible different routes from Raj’s house to Matt’s.
Get Ready Mentally

1. Is each answer greater than (>), less than (<), or equal to 1? Explain how you know.
   a) 0.5 + 0.43
   b) 1.2 − 0.3
   c) $\frac{3}{4} + \frac{1}{2}$
   d) 0.2 + $\frac{1}{2}$ + 0.3

2. Is the perimeter of each shape greater than, less than, or equal to 1 m? Explain how you know.
   a) [diagram]
   b) [diagram]

Get Ready by Thinking

Choose the most reasonable estimate for each of the following. Share your estimates with a partner. Explain your thinking.

4. The volume of pop in a can is about
   A 250 mL   B 1.2 L   C 350 mL   D 0.1 L

5. The height of a grade 8 student is about
   A 150 cm   B 0.6 m   C 1800 mm   D 2.1 m

6. The thickness of a loonie is about
   A 0.5 cm   B 6 mm   C 0.1 m   D 2 mm

7. The mass of a lock is about
   A 500 g   B 150 g   C 50 mg   D 0.7 kg

8. The fraction of pizza that was eaten is about
   A $\frac{1}{5}$   B $\frac{1}{3}$   C $\frac{1}{4}$   D $\frac{2}{3}$
Get Ready by Exploring

Materials
- centimetre grid paper
- scissors
- BLM Get Ready 8A
  Finger Puppets

9. Maya is designing some finger puppets for the drama club. She needs to cut two pieces of fabric for each puppet.

   a) Determine the area of felt needed for each puppet.
   b) What do you notice about the areas?
   c) What is the total area of felt used for these puppets?

10. For each puppet, Maya starts with a square piece of felt. She then cuts pieces from it and rearranges them. What is the area of the original square?

11. Look at the puppet designs on the worksheet. Show how the designs could be cut and rearranged back to squares. Here are some hints to get you started on the first two puppets.
   A: Cut and move one piece.
   B: Cut and move one piece.

12. a) Draw a square with an area of 25 cm².
    b) Design your own puppet by making two or three cuts and rearranging the pieces. Decorate your puppet. Draw another square to show your solution.

Get Ready by Reflecting

13. How is area different from perimeter?

14. Explain why Maya might want to know the perimeter of her puppets. Would they all have the same perimeter?
2 Convert Fractions, Decimals, and Percents; Perfect Squares and Square Roots

Get Ready Mentally

1. Write each fraction as a percent.
   a) \( \frac{1}{2} \)  
   b) \( \frac{1}{4} \)  
   c) \( \frac{1}{5} \)  
   d) \( \frac{3}{5} \)

2. Write each fraction as a decimal.
   a) \( \frac{1}{4} \)  
   b) \( \frac{1}{5} \)  
   c) \( \frac{1}{10} \)  
   d) \( \frac{3}{10} \)

3. Write each decimal as a percent.
   a) 0.25  
   b) 0.45  
   c) 0.9  
   d) 0.333

4. a) What does percent mean? 
   b) Explain how to write a percent as a decimal.

5. Name three decimal numbers that are less than \( \frac{1}{2} \).

Get Ready by Thinking

Origami is an ancient art. Square pieces of paper can be folded to form many different designs.

6. What is the area of a square piece of origami paper with each side length?
   a) 4 cm  
   b) 8 cm  
   c) 100 mm

7. Ashton is organizing his origami paper. What is the side length of a square piece with each area?
   a) 36 cm²  
   b) 4 cm²  
   c) 121 cm²

8. What is the perimeter of a square piece of origami paper with each area?
   a) 16 cm²  
   b) 25 cm²  
   c) 49 mm²

9. To make a paper crane, Kayle folds a piece of origami paper in half twice.
   a) What fraction of the original is the new square that is formed?
   b) Write your answer to part a) as a decimal and as a percent.

10. Shawana got 15 out of 20 marks for her design in an origami contest. Adwin got 19 out of 25 for precise folding. Explain how you could figure out who got the better mark.
Get Ready by Exploring

Elsa created a game called Math Spin for the Math Expo at school. To win, a player must spin the letters of the word MATH in the correct order.

11. If you spin the spinner once, what is the probability that you will get each letter? Express each answer as a fraction and as a percent.
   a) a consonant
   b) the letter T

12. Aaliyah and Michael played the game. Their results are shown in the table.

<table>
<thead>
<tr>
<th>Spin Number</th>
<th>Aaliyah</th>
<th>Michael</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>M</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>4</td>
<td>H</td>
<td>A</td>
</tr>
<tr>
<td>5</td>
<td>T</td>
<td>H</td>
</tr>
<tr>
<td>6</td>
<td>H</td>
<td>T</td>
</tr>
</tbody>
</table>

   a) Who won the game? How many spins did it take?
   b) What is the minimum number of spins needed to win a game of Math Spin?

13. a) Play Math Spin with a partner for 5 min. Record the results of each game, as well as the number of spins it took for someone to win, using a table.
   b) Determine the mean, median, and mode of the number of spins to win the game. Show your calculations.

14. Gather the game results from the whole class. Use them to repeat question 13b) for the whole class.

Get Ready by Reflecting

15. Is Math Spin a fair game? Explain how you know.

16. Describe how you could change this game so that it could be won in fewer spins.
Get Ready Mentally

1. List the next three numbers in each pattern.
   a) 4, 9, 14, ...
   b) –2, –5, –8, ...
   c) 1, 1, 2, 3, 5, ...
   d) 4, 2, 0, ...
   e) 1, 4, 3, 6, 5, ...
   f) –15, –11, –7, ...

2. List the next three numbers in each pattern.
   a) \( \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \ldots \)
   b) \( \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{1}{4}, \ldots \)
   c) \( 3 \frac{1}{2}, 3, 2 \frac{1}{2}, \ldots \)
   d) \( \frac{1}{2}, –2, \frac{1}{3}, –3, \frac{1}{4}, \ldots \)

Get Ready by Thinking

3. In each box, which number does not belong? Explain why.
   a) 4 8 1
      36 49 25
   b) 2 11 13
      5 3 6
   c) 2 4 3
      6 12 9
      6 10 7
      15 30 21

4. Each player at a chess tournament shakes hands once with every other player.
   a) Copy and complete the table.
      
      | Number of Players | Number of Handshakes |
      |-------------------|----------------------|
      | 1                 | 0                    |
      | 2                 | 1                    |
      | 3                 | 3                    |
      | 4                 |                      |
      | 5                 |                      |

   b) Describe the pattern that relates the number of handshakes to the number of players.
   c) How many handshakes would there be for 6 players? 10 players?
   d) Describe the strategy you used to answer this question.

5. Squares can be divided into smaller squares using lines.
   a) How many lines do you need to draw to get 36 smaller squares?
   b) How many smaller squares will you get with 12 lines?

6. Describe what the number cruncher is doing to each set of numbers.

<table>
<thead>
<tr>
<th>a)</th>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>–3</td>
<td>–1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b)</th>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>
Get Ready by Exploring

Ms. King asks each of her students to design a personal tile. The tiles will be joined together with clips at each vertex so that they can be displayed in the hallway.

Ms. King arranged the tiles like this:

1 tile: 6 clips
2 tiles: 6 clips
3 tiles: 6 clips

7. a) On triangle dot paper, sketch the arrangements for four to eight tiles, making sure to show the clips.
   b) Copy and complete the table to show your results from part a).

<table>
<thead>
<tr>
<th>Number of Tiles</th>
<th>Number of Clips</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>26</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>38</td>
</tr>
<tr>
<td>8</td>
<td>44</td>
</tr>
</tbody>
</table>

8. How many clips would Ms. King need for 12 students? Show two different ways to solve this.

9. a) Tiles cost $0.65 each and clips cost $0.07 each. How much would it cost to create a tile display for a class of 25 students?
   b) Suggest another way to arrange the 25 tiles to save some money. Draw your arrangement on another piece of triangle dot paper.

10. a) What is the cost of your arrangement from question 9b)?
    b) Describe some advantages and disadvantages of using your arrangement.

Get Ready by Reflecting

11. Describe a pattern that you have seen. How are the parts related? Explain using pictures, words, and numbers.

12. Write a paragraph about a real-life situation where you used a pattern to help solve a problem.

Materials
- triangle dot paper
Measurement
- Interpret and evaluate measurement formulas.
- Define and measure radius, diameter, and circumference and explain their relationships.
- Develop the formulas for circumference and area of a circle.
- Estimate and calculate the radius, diameter, circumference, and area of a circle.
- Draw a circle given its area and/or circumference.

Geometry and Spatial Sense
- Construct a circle from various information.

Number Sense and Numeration
- Perform multi-step calculations, using calculators, and check them by estimation.
- Express repeated multiplication as powers.

**Key Words**
- radius \( r \)
- diameter \( d \)
- circumference \( C \)
- pi \( \pi \)
Measurement and Number Sense

Imagine you are strapped into this midway ride. As it spins faster and faster, your body presses against the back of the drum. You feel giddy and dizzy.

Why do many people think this ride is so much fun? How many times do you think the ride spins during the ride? How far do you think you travel during the ride?

In this chapter, you will develop formulas for circumference and area of a circle. You will estimate and calculate the radius, diameter, circumference, and area of a circle and draw a circle given its area or circumference.

Chapter Problem

How far will you travel each time the drum spins around once? What information do you need to answer this question?

Think about other circular midway rides you have ridden on. In this chapter, you will learn how to calculate circular measures for these rides.
Rounding With Units

To round a number, look at the digit to the right of the place value to which you are rounding. If the digit is 5 or greater, round up.

44.09 m rounded to the nearest tenth of a metre is 44.1 m.

- Look at the hundredths digit. 9 is greater than 5, so round the previous digit up.

13.2 cm rounded to the nearest centimetre is 13 cm.

- Look at the tenths digit. 2 is less than 5, so leave the previous digit as is.

1. Round to the nearest millimetre (mm).
   a) 25.6 mm  
   b) 4.336 mm

2. Round to the nearest hundredth of a kilometre (km).
   a) 2.485 km  
   b) 0.3109 km

3. Round to the nearest centimetre (cm).
   a) 37.605 cm  
   b) 0.89 cm

4. Round to the nearest tenth of a metre (m).
   a) 0.57 m  
   b) 1.27 m

Working With Ratios

A ratio is a comparison of two or more quantities with the same units. Ratios can appear in various forms. Suppose that your team wins 3 out of 4 of its games. The ratio 3 to 4 can be written as follows:

- 3:4, in ratio form
- $\frac{3}{4}$, in fraction form
- $3 \div 4$, in division statement

5. A candy bag has 6 red and 4 blue jellybeans. Write the ratio of red to blue jellybeans in three different forms.

6. Rewrite each ratio.
   a) $\frac{5}{6}$, in ratio form
   b) $2 \div 5$, in fraction form
   c) $\frac{5}{6}$, as a division statement
   d) 5:7, in fraction form
   e) $2 \div 13$, in ratio form
Squaring a Number

The square of a number is the value of the number multiplied by itself.

\[ 5^2 = 5 \times 5 = 25 \]

Scientific and many graphing calculators have an \( x^2 \) button.

7. Evaluate.
   a) \( 3^2 \)  
   b) \( 10^2 \)  
   c) \( 2.5^2 \)

8. Evaluate.
   a) \( 2.7^2 \)  
   b) \( 0.3^2 \)  
   c) \( 35^2 \)  
   d) \( 100^2 \)

Applying Area Formulas

**Area** is a measure of the space covered by a two-dimensional shape. The table gives the area formulas of some common shapes.

What is the area of this shape?

4.5 cm

8.0 cm

Apply the formula for the area of a rectangle:

\[ A = l \times w \]

Substitute \( l = 8.0 \) and \( w = 4.5 \).

\[ A = 8.0 \times 4.5 \]

Substituted numbers are shown in red.

\[ A = 36 \]

The area of the rectangle is 36 cm\(^2\).

That's about the size of a business card.

9. Find the area of each shape.
   a)
   b)

10. Find the area of each shape.
   a)
   b)
Circles come in many different sizes. Every circle has a **radius**, a **diameter**, and a **circumference**.

**Materials**
- string
- scissors
- ruler
- paper and pencil
- circular objects

**Optional:**
- BLM 1.1A Discover the Pi Relationship

**Is there a constant relationship among measures of circles?**

1. Choose five circular objects. Measure around the outside of each object. What measurement is this?
   - Place a piece of string around the outside of the object. Mark the length around the object. Measure the length of the string.

2. Measure the maximum width of each object. What measurement is this?
   - Trace around the object to make a circle on a piece of paper. Cut out the circle and fold it in half. Measure across the fold.
3. Measure from the centre to the edge. What measurement is this?

Fold the circle in half again. Measure across the fold.

4. Record your measurements in a table like the one below.

<table>
<thead>
<tr>
<th>Radius, $r$ (cm)</th>
<th>Diameter, $d$ (cm)</th>
<th>Circumference, $C$ (cm)</th>
<th>$C + d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>6.0</td>
<td>18.8</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Reflect Look for any patterns or relationships.
   a) Describe the relationships between $C$ and $d$, $d$ and $r$, and $C$ and $r$.
   b) Are these relationships constant? Explain.

Key Ideas

- The circumference of a circle is approximately three times its diameter.
- The ratio of the circumference to the diameter is constant for any circle. This constant value is called $\pi$, often written as the symbol $\pi$.
- The value of $\pi$ is approximately 3.14.

Communicate the Ideas

1. a) How many different ways can you measure a circle? What are these measures called?
   b) The perimeter of any shape is the distance around its outside. How are the perimeter and circumference of a circle related?

2. If you increase the diameter of a circle, what happens to
   a) the circumference?
   b) the ratio of $C + d$?

Writing Ratios

$\frac{C}{d}$ can also be written as $C + d$.
4. State the radius and the circumference of each circle.

3. State the diameter and the circumference of each circle.

5. Use string and a ruler to measure the diameter and the circumference of each circle.

6. Calculate the circumference-to-diameter ratio for each circle in question 5.

7. Without measuring, what is the radius of each circle in question 5?

8. Use string and a ruler to measure the radius and circumference of each circle.

9. Calculate the circumference-to-radius ratio for each circle in question 8.

10. Without measuring, what is the diameter of each circle in question 8?

**Apply**

11. When you measure \(C + d\) for various circles, you might not always get exactly the value of \(\pi\). Suggest reasons for this.

12. Find five circular objects in your classroom or at home.

   a) Describe a method for measuring the radius of each object.

   b) Use your method to measure each radius.

   c) Determine the ratio of circumference to radius for each circle by dividing \(C + r\).

   d) Compare your results with your prediction. Describe what you notice.
13. The playing area of a CD does not go all the way to the centre.

   ![CD Image]

   a) What is the ratio of the outside and inside radii?
   b) How many times longer than the inside “track” of a CD is the outside “track”? Explain your reasoning.

14. Babylonian, Greek, and Egyptian mathematicians all estimated \( \pi \). Research some of these early estimates. Use your local reference library, or go to www.mcgrawhill.ca/links/math8 and follow the links.

15. This dolphin pool has a circumference of 38 m. Estimate its diameter. Justify your choice of estimate.

   ![Dolphin Pool Image]

16. A circular garden has a pathway running through its centre. The pathway is 6.3 m long.

   ![Pathway Image]

   a) What is the radius?
   b) Explain how you could determine the length of the fence around the garden.
   c) Use your method to calculate the length of fence. Round your answer to the nearest tenth of a metre.

**Extend**

17. Soup cans usually have the same shape.

   ![Soup Can Image]

   a) Predict which will be greater, the height of a soup can or its circumference.
   b) Test your prediction using an actual soup can. Compare your findings with classmates.
   c) Discuss reasons why the wrong prediction might be made in part a).

---

**Did You Know?**

Archimedes was the Greek mathematician who first recorded an estimate of \( \pi \). He used polygons to show that \( \pi \) is between \( \frac{223}{71} \) and \( \frac{22}{7} \).
Mousetrap-racers are a fun way to learn about design, motion, and math.

How far will a mouse-trap racer go if its wheels turn eight times? How does the size of the wheels affect the answer?

Discover the Math

What formulas relate to circumference?

1. On a blank sheet of paper, draw a circle with diameter of your choice.

2. a) Explain how you can use the $C + d = \pi r$ relationship to find the circumference of this circle. Do not measure the circumference!
   
   b) Use your method to calculate the circumference.
   
   c) Measure the circumference.
   
   Compare your answers to a) and b). Describe what you notice.
   
   d) Does your calculation method work? If not, try to improve it.
3. a) Use your method to calculate the circumference of a circle with a diameter of 18.0 cm.
   b) Compare your answer with those of your classmates.

4. Reflect Explain how to find the circumference of a circle
   a) if you know its diameter
   b) if you know its radius

Example 1: Diameter and Circumference Formula
A “Pizza-Plus” has a thin string of cheese baked inside the crust. The string of cheese goes around the circumference of the pizza. What length of cheese is stuffed into the crust of the pizza? Round to the nearest centimetre.

Solution
Use the formula $C = \pi \times d$.
The diameter is 30 cm.

\[
C = \pi \times d \\
C = \pi \times 30 \\
C = 94.247... \\
C \approx 94
\]

Each pizza needs about 94 cm of cheese.

This means the circumference is a bit more than 3 times the diameter.

This is about the width of a single bed.
Example 2: Radius and Circumference Formula

Alysia is designing a mousetrap-racer for her science class.

a) Determine the circumference of each wheel. Round to the nearest centimetre.

b) How far will the mousetrap-racer go if the wheels spin eight full turns? Round to the nearest tenth of a metre.

Solution

a) Use the formula \( C = 2 \times \pi \times r \).
   
   The radius of each wheel is 6 cm.
   
   \[
   C = 2 \times \pi \times 6 = 37.699...
   \]
   
   \( C \approx 38 \)

   The circumference of each wheel is about 38 cm.

b) Method 1: Calculate the Distance
   
   The wheels will spin 8 times.
   
   Multiply to get the total distance.
   
   distance = \( 8 \times 38 \)
   
   \[ = 304 \]

   The total distance is 304 cm.

   Convert to metres.

   \[ 304 \text{ cm} = 3.04 \text{ m} \]

   The mousetrap-racer will travel approximately 3.0 m.

Method 2: Measure the Distance

- Take a single CD. Make a mark at one point on the circumference.
- Roll the CD carefully along a tape measure. Start with the mark at 0.

- Record the distance after eight complete rolls. Round to the nearest tenth of a metre.

The mousetrap-racer will travel approximately 3.0 m.
Key Ideas

- If you know the diameter of a circle, you can calculate the circumference. Use the formula \( C = \pi \times d \).
- If you know the radius of a circle, you can calculate the circumference. Use the formula \( C = 2 \times \pi \times r \).

Communicate the Ideas

1. a) What formula could you use to calculate the circumference of this circle? Explain why.
   b) What other formula might you use? What would you have to do before using this formula?

2. a) What formula could you use to calculate the circumference of this circle? Explain why.
   b) What other formula might you use? What would you have to do before using this formula?

3. Suggest a method for finding the diameter of a circle if you know its circumference.

Check Your Understanding

Practise

Remember to include units of measure with all final answers. For help with questions 4 to 7, refer to Example 1.

4. Find the circumference of each circle. Round to the nearest centimetre.
   a) \( 12 \text{ cm} \)
   b) \( 6 \text{ cm} \)

5. A string of cheese goes around the circumference of a pizza. The pizza has a diameter of 20 cm. What length of cheese is stuffed into the pizza? Round to the nearest centimetre.

6. A string of cheese goes around the circumference of a pizza. The pizza has a diameter of 43.5 cm. What length of cheese is stuffed into the pizza? Round to the nearest tenth of a centimetre.
7. Find the circumference of a basketball hoop with diameter 0.5 m. Round to the nearest tenth of a metre.

For help with questions 8 and 9, refer to Example 2.

8. Find the circumference of each circle. Round to the nearest centimetre.
   a) [Image: Circle with diameter 8 cm]
   b) [Image: Circle with diameter 19 cm]

9. What is the circumference of a ring with radius 12 mm? Round to the nearest millimetre.

10. Find the circumference of each circle. Round to the nearest tenth of a metre.
   a) [Image: Circle with diameter 0.9 m]
   b) [Image: Circle with diameter 2.5 m]
   c) [Image: Circle with diameter 3.3 m]
   d) [Image: Circle with diameter 4.7 m]

Apply
For questions 11 to 17, round your answers to the nearest unit.

11. A spoke of a bicycle wheel is 16 cm. What is the circumference of the wheel?

12. This midway ride rotates, spinning the passengers strapped in around its edge. How far do passengers travel each time the ride spins once? Round to the nearest metre.

13. Here is a measurement problem, and Claudia’s solution:
   The diameter of a circle is 12 cm. Find the circumference.
   Solution
   \[ C = 2 \times \pi \times r \]
   \[ C = 2 \times \pi \times 12 \]
   \[ C = 75 \]
   The circumference is 75 cm.
   What is wrong with Claudia’s solution? How can she fix it?

14. Refer to Example 1. Ivan’s favourite part of a “Pizza Plus” is the cheese-filled crust. His mother says that he can order either a large pizza or two small pizzas for himself and two friends.
   a) Which option, if either, gives Ivan more stuffed cheese?
   b) What else might affect Ivan’s decision? Explain.
15. Refer to Example 2. Mateo and Yvonne are using old records for their mousetrap-racers. Mateo is using LPs, which have a diameter of 30 cm. His racer's wheels turn 5 times. Yvonne is using 45s, which have a diameter of 17.5 cm. Her racer's wheels turn 8.5 times. Whose racer travels the farthest?

16. The Saturn V moon rocket consisted of several stacked circular stages, of different diameters. The diagram shows these diameters.

a) Name an object that is about 10 m long.
b) Find the circumference of each stage.
c) Why would an engineer need to know the circumference of each cylinder?

17. A semicircular outdoor theatre is being planned.

Each seat is to be a 0.5-m length of curved bench. How many more people can sit in the back row than in the front row? Describe how you found your answer.
Lac a l’Eau Claire in Québec was formed around 300 million years ago when meteorites crashed to Earth. The photo shows the two circular craters that form the bed of this 20-m deep lake. Both craters have about the same diameter. About how much land does Lac a l’Eau Claire cover?

**Discover the Math**

**How do you calculate the area of a circle?**

1. Cassie drew a circle with radius 5 cm. She cut it up to estimate its area. What are the dimensions marked? Explain your answer.

2. Cassie then put the parts of her circle together, like this.
   a) Does Cassie’s new shape have the same area as her original circle? Explain.
   b) What figure is closest to Cassie’s new shape? How do you calculate the area of that shape?
   c) Use your answer to step 2b) to estimate the area of Cassie’s circle.

3. **Reflect** Can you use Cassie’s trick with any circle?
   a) Write an expression for the horizontal arrow in this diagram.
   b) Write a formula for the area of a circle with radius $r$.
   c) Explain whether you think your formula is approximate or exact.
Example 1: Calculate Area From Radius

Inga is planning to add a circular skylight to her home. She measures a radius of 1.5 m for the skylight. What will the area of the skylight be? Use this formula: \( A = \pi \times r^2 \). Round to the nearest tenth of a square metre.

**Solution**

\[
A = \pi \times r^2 \\
A = \pi \times (1.5)^2 \\
A = \pi \times 2.25 \\
A = 7.068...
\]

\( A \approx 7.1 \)

Inga’s skylight will have an area of approximately 7.1 m\(^2\).

Example 2: Calculate Area From Diameter

Lac a l’Eau Claire is formed by two craters that are roughly circular. Each crater measures about 3600 m across. How much area does each crater cover?

- **a)** in square metres?
- **b)** in square kilometres?

**Solution**

a) The diameter is 3600 m. The radius is half the diameter.
   \[
   r = 3600 \div 2 \\
r = 1800
   \]
   Apply the area formula.
   \[
   A = \pi \times r^2 \\
A = \pi \times 1800^2 \\
A = \pi \times 3240000 \\
A = 10178760.2
   \]
   \( A \approx 10000000 \)
   Each crater covers approximately 10 000 000 m\(^2\).

b) The radius is 1800 m. Convert the radius to kilometres.
   \[
   1800 \text{ m} = 1.8 \text{ km}
   \]
   Apply the area formula.
   \[
   A = \pi \times r^2 \\
A = \pi \times (1.8)^2 \\
A = \pi \times 3.24 \\
A = 10.2
   \]
   Each crater covers approximately 10.2 km\(^2\).
The area of a circle is about 3 times the square of its radius. Use this value to estimate the area of a circle before calculating it.

To calculate the area of a circle, use the formula $A = \pi r^2$.

1. **Communicate the Ideas**

   a) What kind of units are used to measure area?
     b) Why are these units used?

   2. Explain how you can calculate the area of this circle.

   3. What value should you get if you divide the area of any circle by the square of its radius? Explain.

   **Check Your Understanding**

   **Practise**

   *For help with questions 4 to 6, refer to Example 1.*

   4. Find the area of each circle. Round to the nearest square millimetre.
      a) $7 \text{ mm}$
      b) $2.5 \text{ mm}$

   5. A circular skylight has a radius of $0.8 \text{ m}$. What is the area of the skylight? Round to the nearest tenth of a square metre.

   6. A circular porthole has a radius of $15 \text{ cm}$. What is the area of the porthole? Round to the nearest square centimetre.

   **Did You Know?**

   The Canadian Shield is 500 million years old. It is made of Earth’s oldest surface rock. That makes Canada one of the best places on Earth to find ancient craters.

   **For help with questions 7 to 9, refer to Example 2.**

   7. The Manicougan Crater in northern Québec is roughly circular. It measures 100 km across. How much area does it cover, in square kilometres?

   8. Brent Crater in Algonquin Park, Ontario, is roughly circular. It measures approximately 4100 m across. How much area does it cover?
      a) in square metres?
      b) in square kilometres?
9. Find the area of each circle. Round to the nearest tenth of a square centimetre.

a) ![4 cm circle]

b) ![6.5 cm circle]

10. Find the area of each circle. Round to the nearest square unit.

a) ![3 cm circle]

b) ![11 mm circle]

c) ![7 m circle]

d) ![17 cm circle]

Apply

11. A circular skating rink has a diameter of 28 m. How much ice area is available for skaters?

12. This midway ride lies flat when it is not in operation. Suppose you need to make a rain cover for this ride. How big should the cover be? Use numbers, words, and pictures.

13. Nadia is playing golf. She estimates the diameter of a circular golf green at about 2.5 m. What is the area of the green? Round to the nearest square metre.

14. a) Draw a circle on centimetre grid paper. Use the grid squares to estimate the area of the circle.

b) Measure the radius of the circle. Calculate the area of the circle, using the radius-area formula.

c) Compare your answers to parts a) and b). How close were they?

d) How could you improve your estimate from part a)?

Extend

15. The distance from the centre of the bulls-eye to the outer ring of a dart board is 17 cm. You can score points for any dart landing inside the outer ring.

a) How large is the scoring area? Round to the nearest square centimetre.

b) In one type of dart game, you must "double-in" to start collecting points. This means that your dart must land within a narrow band called the "double ring." How large is the area of the double-ring section?

Circles are found in many places and often have many different meanings. They can be used to represent life cycles, seasons, and orbits. Some cultures use circles in their rituals and dances. How have you seen circles used in
- games and sports?
- artistic designs?

The photo shows a fabric design containing circles. How do you think the artist might have constructed the circles?

How do you construct a circle using a set of compasses?

**Example 1: Construct a Circle Given the Centre and the Radius**

Draw a circle with a radius of 3 cm.

**Solution**

Draw a line segment 3 cm in length. Choose one endpoint and let it be the centre of the circle.
Example 2: Construct a Circle Given the Centre and a Point on the Circle

Draw a circle with centre A and passing through point B.

Solution

Copy the points A and B onto a sheet of paper.

Set the point of your compasses at the centre A. Adjust the compasses so that the circle will pass through point B.

Construct the circle.
Key Ideas

- To construct a circle, given the centre and the radius,
  - set compasses to the measure of the radius
  - centre the compasses at the given centre
  - construct the circle

- To construct a circle, given the centre and one point on the circle,
  - centre the compasses at the given centre
  - set the compasses so that the circle will pass through the point
  - construct the circle

Communicate the Ideas

1. Use a set of labelled diagrams to show how to draw a circle with a radius of 4 cm.

2. Describe how you would construct a circle with centre C and passing through point P.

Check Your Understanding

Practise

For help with questions 3 and 4, refer to Example 1.

3. Draw a circle with each radius.
   a) 5 cm
   b) 30 mm
   c) 4 cm

4. Construct a circle with each radius.
   a) __________________________
   b) __________________________
   c) __________________________

For help with questions 5 and 6, refer to Example 2.

5. Copy points A and B. Draw a circle with centre A passing through point B.
   a) __________________________
   b) __________________________
6. Copy points C and D. Draw a circle with centre C passing through point D.
   a) 
   
   b) 

Apply
7. a) Copy this diagram on centimetre grid paper.

b) Construct a circle of radius 5 square widths. Centre it on point S.
   c) Which points fall inside the circle? What word do they spell?

8. a) Copy this diagram on centimetre grid paper.

b) Construct a circle with centre A. The circle should pass through point B.

9. a) Draw a circle with a radius of 3 cm.
   b) Find the circumference of this circle. Round to the nearest centimetre.
   c) Find the area of this circle. Round to the nearest square centimetre.

10. a) Copy this diagram on centimetre grid paper.

b) Construct all the circles you can, using pairs of points for centres and edge points. Describe any patterns you see.

11. Create a design using only circles with radii of 4 cm, 5 cm, and 7 cm. Colour your design.

Literacy Connections

Plural Forms
The plural form of radius is radii.

12. a) Draw a circle with a radius of 2.5 cm.
   b) Find the circumference and the area of the circle.
   c) Predict what happens to the circumference and the area of the circle if you double the radius.
   d) Draw the new circle.
   e) Find the circumference and the area of the new circle.
   f) Were your predictions correct? Explain.
Many parks have a children's play area. How can you use math to find the best place to locate it?

**Discover the Math**

How do you construct a circle to fit geometric data?

Laurie, an environmental engineer, is working on some projects for a provincial park. One of her tasks is to locate a children's playground. Her planning team agrees that the playground should be the same distance from each of three camping areas, shown on the map.

1. **a)** Make a copy of the map on a sheet of paper. Draw strongly enough that you can see your drawing through the back of your paper. Lightly mark the location on your map.

2. Laurie determines that the playground should be the centre of a circle that passes through the three campgrounds. Why is this true?
3. a) Fold your map so that Aspen Camp and Birch Camp exactly line up. Make a good crease.
   b) Unfold your map. Repeat step a) for Aspen Camp and Cedar Camp.
   c) Unfold your map. Mark the playground site where the creases cross each other. How close is this to your estimated position?

4. You need to check that the playground site you constructed is correct.
   a) Set the point of your compasses on the playground site.
   b) Adjust the pencil so that it passes through Aspen Camp.
   c) Complete the circle. How close does it come to the other two camps?

5. Reflect For steps 1 to 4, describe
   a) the geometric data you were given
   b) how you used the data to construct a circle

---

**Key Ideas**

To construct a circle, given three points on the circle,
- fold the paper so that one pair of points line up, and crease the fold
- unfold the paper and repeat with a different pair of points
- unfold the paper and centre the compasses at the constructed centre
- set the compasses so that the circle will pass through any of the points
- construct the circle, checking that it goes through all three points

**Communicate the Ideas**

1. Explain how you can find the centre of a circle if you know three points around the circumference.

2. After finding the centre of a circle using paper folding, Maria suggested another method. Is she correct? Explain how you know.

I found the centre a different way. I joined the points to form two line segments. Then, I used a ruler to measure and mark a point in the middle of each line segment. Then, I drew a line through each middle point crossing each line segment at right angles. The centre of the circle is the point where the two new lines meet.
Practise

3. Copy this diagram on centimetre grid paper.

![Diagram of three points A, B, and C on a grid]

a) Find the centre of the circle that passes through points A, B, and C. Draw the circle.

b) Explain how you found the centre.

4. Copy this diagram on centimetre grid paper.

Find the centre of the circle that passes through the three points shown.

![Diagram of four points U, V, T, and U on a grid]

Apply

5. Stella, Mary, and Raman are writing a journal entry together about using three points to draw a circle. They can’t agree what to say. Who is right, and why? Explain, using examples.

If there are three points, then there is always at least one way to draw a circle through them.

I agree.

Mary

You’re wrong. It won’t always be possible to draw a circle to join three points.

Stella

Raman

Extend

6. a) Copy this diagram on centimetre grid paper.

b) Using line segments AB and AC, apply the paper-folding method to locate a circle that passes through A, B, and C. Draw the circle.

c) Repeat part b) using line segments BC and AC. Describe what you notice.

7. The regional councils of three small towns decide that a water tower should be constructed at a location that is an equal distance from each town. Use the following information to draw a map that shows all three towns, and exactly where the water tower should be built.

- Elmwood is 12 km directly east of Drayden.
- Hargrove is 8 km directly south of Drayden.

8. An archaeologist has discovered the remains of an ancient wheel. Using mathematics, the archaeologist can determine the original size of the wheel.

a) Copy the diagram as accurately as possible.

b) Accurately reconstruct what the wheel looked like.

c) Find the circumference and the area of the wheel.

d) Describe your method.
Construct Circles Using The Geometer's Sketchpad®

In this activity, you will use The Geometer's Sketchpad® to help Laurie locate the children's playground.

Part 1 Model the Park Layout

1. Open The Geometer's Sketchpad® and begin a new sketch.

2. From the Edit menu, choose Preferences. Set the preferences as shown.

3. Click on the Point Tool. Place a point near the bottom left corner of the screen.

4. Click on the Text Tool. Label this point A:
   • Click the point once to label it.
   • Click and drag the label if you want to adjust its position.

5. Click on the Straightedge Tool. Starting from point A, draw a vertical line segment. Label the top point as B.

Materials
- computers
- The Geometer's Sketchpad® software

Alternatives:
- TECH 1.5A Construct Circles (GSP 4)
- TECH 1.5B Construct Circles (GSP 3)

Technology Tip
- Holding the Shift key while dragging makes it easier to draw a vertical segment.
- Before you select a new object, make sure that you deselect first by clicking somewhere in the white space. Then, select the objects you want.
6. a) Choose the Selection Arrow Tool. An arrow will appear. Select points A and B by clicking on them with the arrow.
b) From the Measure menu, choose Distance.
c) Click and drag B until the distance AB equals 5.0 cm. It may be difficult to get the exact length. Try to keep AB perfectly vertical.

7. a) Draw a horizontal line segment from A, going right. Label point C.
b) Measure the distance AC. Click and drag C until this distance equals 3.5 cm.

8. Suppose A, B, and C represent the three camping areas. Using the Point Tool, place a point D approximately where you think the playground should go.

9. Laurie determines that the playground should be the centre of a circle that passes through the three campgrounds. Why is this true?

10. a) Using the Compass Tool, try to draw a circle that passes through points A, B, and C.
b) How does this help you find the position of the playground?
c) Change your placement of point D if you want.

Technology Tip

- To delete an object, select it using the Selection Arrow Tool. Then, press the Backspace or Delete key. You can delete multiple objects at once. If you accidentally delete an object, select Undo Delete Objects from the Edit menu.
Part 2: Construct a Circle to Locate the Playground

Use your sketch from Part 1. Delete point D and any circles.

1. Select line segment AB. From the Construct menu, choose Midpoint. Label the midpoint D.

2. Select the midpoint and line segment AB. From the Construct menu, choose Perpendicular Line. A line should appear. This line divides the line segment AB exactly in half, crossing it at right angles. Deselect all these objects before you begin the next step.

3. Construct the midpoint E on AC. Repeat step 2 for the midpoint and line segment AC.

4. a) Select the perpendicular lines from steps 2 and 3. From the Construct menu, choose Intersection. Label this point P.
   b) Why is point P important to Laurie?

5. a) Select P and A, in that order. From the Construct menu, choose Circle By Centre+Point. Deselect.
   b) Describe what you notice about this circle.

6. a) Construct and then measure the line segments PA, PB, and PC. What do you notice?
   b) How are these distances related to the circle?
   c) Explain how these distances help Laurie place her playground.
   d) What special name can you give to PA, PB, and PC?

7. Suppose one of the campgrounds is moved. How will this change where the playground should go? Click and drag any of the points A, B, and C and describe what you notice.
Designs like this crop circle near Montréal, Québec, have been reported in farms all over the world. How they are created is often unknown. How could you find the diameter of this crop circle, if you knew the circumference?

**Discover the Math**

**Which formula do you use?**

**Example 1: Circumference From Radius**

A child’s wagon wheel has spokes 8 cm long. How far will the wagon travel in one turn of the wheels?

**Solution**

Each spoke connects the centre of the wheel to the rim. So, the radius is 8 cm. Apply the radius-circumference formula:

\[ C = 2 \pi r \]

\[ C = 2 \pi \times 8 \]

Estimate: \( 2 \times 3 \times 8 = 48 \)

\[ C = 50.265\ldots \]

Round to the nearest centimetre.

The circumference is approximately 50 cm. The wagon will travel about 50 cm in one turn of the wheels.
Example 2: Find the Diameter and Radius From the Circumference

The circumference of the large crop circle on the previous page is 120 m.

a) What is the crop circle's diameter, to the nearest metre?

b) What is the crop circle's radius?

Solution

a) \( C = \pi \times d \).
   Multiply \( \pi \times d \).
   Use various possible values for the \( d \) variable.
   Aim for an answer of about 120 m.

\[
\pi \times 40 = 125.663 \ldots \quad \text{Too high.}
\]
\[
\pi \times 38 = 119.380 \ldots \quad \text{Very close.}
\]
\[
\pi \times 39 = 122.522 \ldots \quad \text{Not quite as close.}
\]

To the nearest metre, the closest value is \( d = 38 \).
The large crop circle has a diameter of about 38 m.

b) The radius is half of the diameter.

\[
r = \frac{d}{2} = \frac{38}{2} = 19
\]

The crop circle has a radius of about 19 m.
Example 3: The Cost of a Pathway

A circular garden has an area of 38 m$^2$. A flagstone pathway is being built to run through the centre of the garden. It costs about $12 per metre for the flagstones. Determine the cost of the pathway.

Solution

I must find the cost of the pathway. The pathway runs through the centre of the garden. I need to find the diameter.

1. Use the area to find the radius.
2. Use the radius to calculate the diameter.
3. Determine the cost.

1. Use the area formula to find the radius.
   Use $A = \pi \times r^2$. Try various values for $r$.
   Keep trying until you get an answer close to 38 m$^2$.

   $\pi \times 10^2 = 314$  
   $\pi \times 3^2 = 28.3$   
   $\pi \times 4^2 = 50.3$   
   $\pi \times 3.5^2 = 38.48$  
   $\pi \times 3.4^2 = 36.30$  

   The radius of the circle is 3.5 m.

2. Use the radius to calculate the diameter.

   $d = 2 \times r$  
   $d = 2 \times 3.5$  
   $d = 7.0$

   The diameter is about 7.0 m.

3. Determine the cost.
   Flagstones cost $12 per metre. Multiply by the length needed.
   $12 \times 7 = 84$
   The cost of the flagstones is $84.

The area of the garden is 38 m$^2$. From the formula $A = \pi \times r^2$, $r^2$ is about $38 \div 3$ or 12.7. The value 12.7 is the square of a number between 3 and 4, so the radius is between 3 and 4. So, a diameter of about 7.0 m is reasonable and the cost of $84 is also reasonable.
Summary of circle formulas:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 2\pi r$</td>
<td>radius-circumference</td>
</tr>
<tr>
<td>$C = \pi d$</td>
<td>diameter-circumference</td>
</tr>
<tr>
<td>$A = \pi r^2$</td>
<td>radius-area</td>
</tr>
</tbody>
</table>

- If you are given the circumference of a circle, you can use systematic trial to find the diameter or the radius. Use the formula $C = \pi d$ or $C = 2\pi r$.
- If you are given the area of a circle, you can use systematic trial to find the radius. Use the formula $A = \pi r^2$. Then, double the radius if you need to find the diameter.

Communicate the Ideas

1. The circumference of a bicycle wheel is 150 cm.
   a) Estimate the diameter and the radius.
   b) Explain how you found your estimates.

2. Explain how to find the radius of a circle if you know
   a) its circumference
   b) its area

Check Your Understanding

Practise

3. Find the radius of each circle, to the nearest tenth of a unit.
   a) 15.7 cm  
   b) 40.8 mm

4. Find the diameter of each circle, to the nearest unit.
   a) 125 cm  
   b) 58 m
5. Find the radius of each circle. Round to the nearest tenth of a unit.

\[ \text{a)} \quad \text{Area} = 60 \text{ m}^2 \]

\[ \text{b)} \quad \text{Area} = 100 \text{ cm}^2 \]

6. Find the diameter of each circle. Round to the nearest unit.

\[ \text{a)} \quad \text{Area} = 512 \text{ m}^2 \]

\[ \text{b)} \quad \text{Area} = 6500 \text{ mm}^2 \]

7. A circle has a circumference of 30 cm.
   a) Find the diameter. Round to the nearest tenth of a centimetre.
   b) Draw the circle. Label the diameter.

8. A circle has an area of 120 cm².
   a) Find the radius. Round your answer to the nearest tenth of a centimetre.
   b) Draw the circle. Label the radius.

Apply

For help with question 9, refer to Example 1.

9. A small wagon wheel has spokes 12 cm long. How far will the wagon travel in one turn of the wheels?

For help with question 10, refer to Example 2.

10. A crop circle has a circumference of 87 m.
    a) What is the crop circle’s diameter, to the nearest metre?
    b) What is the crop circle’s radius?

For help with question 11, refer to Example 3.

11. A circular garden has an area of 15 m². Cameron wants to build a flagstone pathway across the diameter of the garden. The flagstones he has chosen cost $15 per metre. What is the cost of the pathway?

12. A bicycle has spokes 16 cm long. How far will the bicycle travel in two turns of the wheels?

13. Marissa has two circular wooden tables. She wants to paint the top of each table. The radius of each table is 30 cm. What area will she need to paint?

Chapter Problem

14. A ferris wheel takes riders from ground level to a height of 20 m.

   a) How far do they travel in one circle?
   b) How far do they go in 10 circles?

15. A circular park has an area of 5000 m².
    a) What length of fence is needed to surround the park?
    b) What will it cost to build the fence if fencing costs $25 per metre?
16. A radio station claims that its signal reaches a broadcast area of 32,000 km².
   a) How far can you drive away from the transmitter before losing the signal? Round your answer to the nearest 10 km.
   b) Describe how you solved this problem.

17. To estimate the area of his circular garden, Bulwinder walks around the circumference. He walks 16 paces and estimates each pace is 1 m.
   a) What does Bulwinder’s walk reveal about the size of the garden? Find as many measures as you can. Show your work.
   b) Can you find the area of Bulwinder’s garden? Explain how.

18. Shi-Anne’s cottage is on the shores of Circle Lake. Shi-Anne’s cottage and Sonja’s cottage are on opposite sides of this circular lake. It is a 2.5-km walk along the shoreline between the cottages.
   a) How far must Shi-Anne swim across the lake to get to Sonja’s? Use pictures and words to show how you know.
   b) In one hour, Shi-Anne can swim 1.5 km, and walk 3 km. Which way is faster? Justify your answer.

19. Maria and her younger brother Chico are racing at a circular park with a circumference of 250 m. Because Maria can run twice as fast as Chico, they agree on these rules.
   - Maria will run once along the path around the circumference of the park.
   - Chico will run along the diameter and back.
   a) Is this fair? Justify your answer with appropriate calculations and diagrams.
   b) What method did you use to solve this problem?
   c) Describe another method you could use.

20. a) Create a circle problem involving area, circumference, radius, and/or diameter.
    b) Solve the problem.
    c) Trade with a partner and solve each other’s problem.
    d) Describe how you could modify your problem to make it more challenging.

22. Bippy the hamster runs 42 m every morning on his hamster wheel. In the course of Bippy’s workout, his wheel spins 75 times. Determine the circumference and radius of Bippy’s hamster wheel, to the nearest tenth of a centimetre. Explain your method.

23. The second hand on a clock extends from the centre of the clock to the numerals. The tip of the second hand travels 2 cm every second.
   a) What is the length of the second hand?
   b) Explain how you solved this problem.
Key Words

Copy and complete each statement.

1. The ______ of a circle can be calculated using the formula \( C = \pi r^2 \).

2. The ratio \( \frac{\text{circumference}}{\text{diameter}} \) is equal to \( 2\pi \).

3. To calculate the ______ of a circle from its area, find the ______, and then multiply by 2.

1.1 Discover the Pi Relationship, pages 12–15

4. Use string and a ruler to measure the circumference and diameter of each circle.

5. a) Predict the ratio \( \frac{C}{d} \) for each circle in question 4.
   b) Calculate \( \frac{C}{d} \) for each circle in question 4. Compare the answer with your prediction.

1.2 Circumference Relationships, pages 16–21

6. The spokes on Jesse's unicycle wheel are 0.4 m long.
   a) What is the circumference of the wheel? Round your answer to the nearest tenth of a metre.
   b) Jesse wants to travel 10.0 m. How many revolutions must he remain balanced for?
   c) Explain how you solved parts a) and b).

7. A circular swimming pool has a circumference of 50 m. How far is it to swim straight across, through the centre? Round your answer to the nearest metre.

1.3 Discover the Area of a Circle, pages 22–25

8. The Zamboni® 700 ice resurfacer has a 244-cm spinning blade. How much area does the blade shave each time it spins?

9. Ryan is buying a pool cover for his swimming pool. What area needs to be covered? Round your answer to the nearest square metre.

1.4 Draw Circles Using a Set of Compasses, pages 26–29

10. Draw a circle with each radius.
    a) 5.5 cm
    b) 37 mm

11. Construct a circle with each radius.
    a) ________________
    b) ________________
12. Copy this diagram on centimetre grid paper. Construct a circle with centre A and a radius of 4 square widths. Does point B lie inside or outside the circle?

13. Copy this diagram on centimetre grid paper.

a) Find the centre of the circle that passes through points P, Q, and R. Draw the circle.

b) Explain how you found the centre.

14. Three towns are located as shown on the grid. Suppose you would like to build a gas station that is an equal distance from each town.

a) Copy the diagram on a sheet of grid paper.

b) Find the location for the gas station.

c) Explain why this point is at the centre of a circle that passes through all three towns.

1.5 Construct Circles From Given Data, pages 30–32

15. A sprinkler can spray water a maximum distance of 8 m in all directions. What area of lawn can this sprinkler water? Round your answer to the nearest square metre.

16. A pathway around a circular garden is 28 m long. Sketch a diagram. Find the diameter, radius, and area of this garden. Round each measure to the nearest unit. Label all measures on your diagram.

17. In pottery class, Marcus plans to make a circular plate with a minimum area of 200 cm². He wonders what diameter he should plan for.

a) Determine the minimum diameter of the plate. Round your answer to the nearest centimetre.

b) Describe each stage of your problem-solving approach to this question.

18. A new “mega-size” pizza costs $29.99. The pizza company wants to get $0.02 per square centimetre of pizza.

a) How much area should the pizza cover?

b) What should be the pizza's diameter, to the nearest centimetre?
**Multiple Choice**

For questions 1 to 4, choose the best answer.

1. The circumference of the circle is
   - A 20 cm
   - B 31 cm
   - C 63 cm
   - D 314 cm

2. The circumference of the Frisbee™ is
   - A 38 cm
   - B 56 cm
   - C 62 cm
   - D 75 cm

3. The ratio \( \frac{\text{circumference}}{\text{diameter}} \) is equal to
   - A \( \pi \)
   - B \( 2 \times \pi \)
   - C \( \pi \times d \)
   - D \( 2 \times \pi \times r \)

4. A circular pen is constructed to hold cattle. The radius of the pen is 18 m. What area do the cattle have?
   - A 57 m²
   - B 113 m²
   - C 324 m²
   - D 1018 m²

**Short Answer**

5. A decorative lunch plate has a diameter of 36 cm. What is its area? Round your answer to the nearest square centimetre.

6. A circular window has a diameter of 3 m.
   a) Trim costs $6.50 per metre. What will it cost to put a trim around the window?
   b) Glass costs $20 per square metre. What will it cost to replace the glass in the window?

7. Bim the cat likes to chase a piece of tinfoil attached to a piece of string. Bim trains his pet human to swing the tinfoil in a circular path with a radius of 1.0 m.
   a) How far does Bim run each lap?
   b) How many laps must Bim run in order to complete his 50-m workout?
   c) What area must Bim have his pet human keep clear of obstacles? Round your answer to the nearest tenth.

8. Draw a circle with this radius.
9. A circle is centred on A and passes through B. What would happen to the circle if
   a) B moved away from A?
   b) A moved toward B?

10. a) Describe how you can construct a circle that passes through three given points.
    b) Think of an example. Use it to show how your method works.

11. Draw a circle with each radius.
    a) 42 mm
    b) 7.5 cm

12. a) What is the radius of a circular swimming pool that has an area of 64 m²? Show your answer to the nearest metre.
    b) How far will you swim if you go across the pool and back once?

Extended Response

13. At a marine-wildlife park, dolphins play in a circular pool with a surface area of 119 m². You can also go downstairs and watch the dolphins through a glass wall.

   a) What length of glass is needed to surround the pool? Round your answer to the nearest metre.
   b) If the viewing windows are 2 m high, what is the total area of glass required? Round your answer to the nearest square metre.
   c) Describe how you solved parts a) and b).

Chapter Problem Wrap-Up

Report on a circular midway ride of your choice.

For information about midway rides, go to www.mcgrawhill.ca/links/math8 and follow the links.

1. Decide and explain what the ride does.

2. Calculate all the circular measures of the ride.

3. Compare your ride to the other rides discussed in this chapter. Describe any similarities and differences.
Geometry and Spatial Sense
- Investigate, explain, and apply the Pythagorean relationship.

Number Sense and Numeration
- Understand that the square roots of non-perfect squares are approximations.
- Estimate square roots without a calculator.
- Find approximate square roots using a calculator.
- Use estimation to justify the reasonableness of calculations.

Measurement
- Interpret and evaluate the use of measurement formulas.

Patterning and Algebra
- Solve first degree equations in one variable by inspection or systematic trial.

Key Words
- Pythagorean relationship
- hypotenuse
- legs (of a right triangle)
- perfect square
- estimate
- approximate
- non-perfect square
Two-Dimensional Geometry

Geometry is an essential part of any construction project. Triangles are used in construction because they provide strength and stability. Right triangles are particularly important in building houses. Builders use a special property of their sides to ensure that walls meet at right angles.

In this chapter, you will explore how the sides of a right triangle are related. You will apply this relationship to solve problems.

Chapter Problem

You are given a piece of wood 100 cm in length. You are allowed to cut the stick of wood once, to make two pieces. How can you use the two pieces of wood to build a kite? If you glue a sheet of kite paper on one side of your frame, what will the perimeter of the kite paper be? What will its area be?

In this chapter, you will explore different possible kites.
Classify Triangles

Triangles can be classified according to the lengths of their sides as well as the types of angles they contain.

- **equilateral triangle**: three equal sides
- **isosceles triangle**: two equal sides
- **scalene triangle**: no equal sides
- **acute triangle**: three acute angles
- **right triangle**: one right angle
- **obtuse triangle**: one obtuse angle

1. Classify each triangle in two ways: by its sides and by its angles.

   a) ![Triangle A]
   
   b) ![Triangle B]

2. How are these two triangles alike? How do they differ?

   ![Triangle C]
   ![Triangle D]

Squares and Square Roots

The **square** of a number is calculated by multiplying the number by itself.

The square of 3 is $3^2$. Its value is $3 \times 3$ or 9. Reversing the process, the **square root** of 9 is 3. This is written as $\sqrt{9} = 3$.

3. What is the value of each square?
   - a) $5^2$
   - b) $7^2$
   - c) $2^2$
   - d) $8^2$

4. What is the value of each square root?
   - a) $\sqrt{4}$
   - b) $\sqrt{25}$
   - c) $\sqrt{49}$
   - d) $\sqrt{100}$
Perimeter and Area of a Square and of a Triangle

The **perimeter** of a shape is the distance around the outside. The **area** is the number of square units of space covered.

- **Square:**
  \[ P = 5 + 5 + 5 + 5 \text{ or } P = 4 	imes 5 \]
  \[ P = 20 \]
  The perimeter of the square is 20 m.

- **Area:**
  \[ A = l \times w \text{ or } A = s^2 \]
  \[ A = 5 \times 5 \text{ or } A = 5^2 \]
  \[ A = 25 \]
  The area of the square is 25 m².

- **Triangle:**
  \[ P = 5 + 12 + 13 \]
  \[ P = 30 \]
  The perimeter of the triangle is 30 m.

- **Area:**
  \[ A = b \times h + 2 \]
  \[ A = 12 \times 5 + 2 \]
  \[ A = 60 + 2 \]
  \[ A = 30 \]
  The area of the triangle is 30 m².

5. Find the perimeter and the area of each square.
   a) 7 m
   b) 11 cm

6. Find the perimeter and the area of each triangle.
   a) 17 cm
   b) 5 m

Solve Equations

Solve for \( x \).

- **a)** \( x + 7 = 12 \)
  \[ x = 5 \text{ By inspection, } 5 + 7 = 12. \]

- **b)** \( 25 = 16 + x \)
  \[ x = 9 \text{ By inspection, } 25 = 16 + 9. \]

7. Solve for \( x \).
   a) \( x + 10 = 18 \)
   b) \( x + 15 = 24 \)
   c) \( 4 + x = 16 \)
   d) \( 12 + x = 25 \)

8. Solve for \( x \).
   a) \( 20 = 9 + x \)
   b) \( 36 = x + 20 \)
   c) \( 32 = x + 16 \)
   d) \( 81 = x + 49 \)
In music, a chord is a combination of three or more notes sounded together in harmony. The notes sound pleasing when played together.

Pythagoras (about 580–500 B.C.E.) was the leader of a group of people called the Pythagoreans. They believed that patterns in whole numbers could be used to explain the universe. They searched to find such patterns in geometry, astronomy, and music.

Pythagoras and this guitarist have something in common. Pythagoras experimented with stringed instruments, looking for ratios of lengths that would produce pleasing sounds when played together. He discovered the octave, and the perfect fourth and fifth chords. These chords are widely used in popular music.

**Materials**
- ruler
- protractor
- centimetre grid paper
- scissors
- tape

**Optional:**
- BLM 2.1A Pythagorean Relationship Recording Sheet

**Discover the Math**

**What is the Pythagorean relationship?**

1. Draw and cut out each set of squares from centimetre grid paper. Write the area, in square centimetres, on each square piece.

   - 3 cm by 3 cm, 4 cm by 4 cm, and 5 cm by 5 cm
   - 5 cm by 5 cm, 12 cm by 12 cm, and 13 cm by 13 cm
   - 6 cm by 6 cm, 8 cm by 8 cm, and 10 cm by 10 cm
   - 4 cm by 4 cm, 5 cm by 5 cm, and 6 cm by 6 cm
   - 4 cm by 4 cm, 12 cm by 12 cm, and 14 cm by 14 cm
2. Make a triangle with each set of three squares. Arrange the three squares so that one side of each square forms a side of the triangle. Tape each arrangement onto a piece of paper.

3. Use a protractor, or the corner of a piece of paper, to determine whether or not any of the triangles contain a right angle.

4. Copy and complete the table based on your observations of the five triangles.

<table>
<thead>
<tr>
<th>Triangle Sides (cm)</th>
<th>Areas of Squares on Sides (cm²)</th>
<th>Type of Triangle (right, acute, or obtuse)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

5. Reflect Look for a pattern in your results.
   a) Consider the first three triangles. Compare the areas of the squares on the sides of each triangle. Write a sentence to describe the relationship among them.
   b) Now, examine the last two triangles. Does the relationship for the first three triangles hold for these two? How are these triangles different from the first three?

You have explored what is called the **Pythagorean relationship**. The sum of the areas of the squares on the two shorter sides, or **legs**, of a right triangle is equal to the area of the square on the **hypotenuse**.

**legs**
- the two shorter sides of a right triangle
- legs meet at 90°

**hypotenuse**
- longest side of a right triangle
- opposite the right angle
Example 1: Find the Area of the Square on the Hypotenuse

A square is drawn on each side of \( \Delta ABC \). Find the area of the square on side AC.

**Solution**

\[ AB^2 + BC^2 = AC^2 \]

\[ 4 + 16 = AC^2 \]

\[ 20 = AC^2 \]

The area of the square on side AC is 20 cm\(^2\).

Example 2: Find the Area of the Square on a Leg of a Right Triangle

A square is drawn on each side of \( \Delta PQR \). Find the area of the square on side PQ.

**Solution**

**Write the Pythagorean relationship.**

\[ QR^2 + PQ^2 = PR^2 \]

\[ 8 + PQ^2 = 13 \]

By inspection, \( 8 + 5 = 13 \).

\[ PQ^2 = 5 \]

The area of the square on side PQ is 5 cm\(^2\).

**Key Ideas**

- The Pythagorean relationship relates the areas of the squares on the three sides of a right triangle.

- The sum of the areas of the squares on the two legs of a right triangle is equal to the area of the square on the hypotenuse.

\[ BC^2 + CA^2 = BA^2 \]
1. What property must a triangle have in order for the Pythagorean relationship to be true? Explain how you came to this conclusion.

2. Does the size of the right triangle affect whether the Pythagorean relationship holds true?

3. Three squares are placed together as shown. Marsha says “Triangle ABC looks like a right triangle to me.” Explain how you know that $\triangle ABC$ is not a right triangle. Use your knowledge of the Pythagorean relationship.

4. What is the area of the square on the hypotenuse of each triangle?

   a) 
   
   b) 

5. Find the area of the square on the hypotenuse of each triangle.

   a) 

   b) 

To answer question 2:
- State your opinion in a sentence.
- Test the Pythagorean relationship on two triangles, one large and one small. Measure side lengths and calculate the area of the square on each side.
- Explain what you find. Use one paragraph for each triangle.
- Finish with a statement of your conclusion.
For help with questions 6 and 7, refer to Example 2.

6. What is the area of the square on the third side of each triangle?
   a) 10 cm²  
      ![Diagram](image)
   b) ![Diagram](image)

7. Find the area of the square on the third side of each triangle.
   a) ![Diagram](image)
   b) ![Diagram](image)

8. In each figure,
   - identify the hypotenuse
   - use the Pythagorean relationship to find the missing area
   a) ![Diagram](image)
   b) ![Diagram](image)

9. In a right triangle, what is the area of the square on the hypotenuse if the areas of the squares on the legs are as follows?
   a) 11 cm² and 15 cm²
   b) 10 m² and 4 m²
   c) 7 cm² and 21 cm²
   d) 26 km² and 13 km²

Apply

10. For which of the following sets of three squares does the Pythagorean relationship hold? Show your work.
   a) 10 cm², 4 cm², 14 cm²
   b) 3 cm², 12 cm², 16 cm²
   c) 5 cm², 27 cm², 22 cm²
   d) 21 cm², 2 cm², 19 cm²

11. The square on the hypotenuse of a right triangle has an area of 34 cm². The square on one of the other sides has an area of 17 cm².
   a) What is the area of the square on the remaining side?
   b) What other way can you classify this right triangle?

12. Does the Pythagorean relationship hold true for a right triangle with side lengths that are not all whole numbers? Give your reasoning and provide examples to illustrate your answer.

13. Larry drew a triangle with side lengths of 5 cm, 7 cm, and 9 cm. Find the areas of the squares on the three sides of the triangle. Is the triangle a right triangle? Explain why or why not.

14. The areas of the squares on two sides of a right triangle are 26 cm² and 40 cm². What are the possible areas for the square on the third side? Draw sketches to support your answers.
15. Kara has several square stickers of four different sizes, as shown.

Kara is curious whether she can place any three of her stickers together to form a right triangle. Solve this problem for her. Use diagrams to show your answer.

**Extend**

16. Does the Pythagorean relationship work with a shape, other than a square, drawn on the sides of a right triangle? Investigate this question with either paper and pencil or The Geometer’s Sketchpad®.

17. The Pythagorean relationship is named after the Greek mathematician, Pythagoras, but it is likely that the Babylonians and ancient Chinese knew the relationship hundreds of years earlier. Research the history of the Pythagorean relationship. Why was Pythagoras given the credit for the relationship?

18. Several Web sites provide proofs of the Pythagorean relationship. Go to [www.mcgrawhill.ca//links/math8](http://www.mcgrawhill.ca//links/math8) and follow the links to find some of the proofs.

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**Making Connections**

A concept map is a graphic representation of properties or relationships that help you organize and clarify information. To create a concept map, you must think about the concepts and how they are related to each other.

Copy and complete this concept map by replacing each ■ with the appropriate number.

**Definition:**

A polygon with ■ sides.

**Right**
- one angle is ■°
- sides are related by the Pythagorean relationship

\[ a^2 + b^2 = c^2 \]

**Equal Sides**
- ■ equal sides
- ■ equal angles
- may be acute or obtuse

**Scalene**

**Equilateral**
- ■ equal sides
- ■ equal angles

**Isosceles**
- ■ equal sides
- ■ equal angles
- may be acute or obtuse

**Right**
- one angle is ■°
- sides are related by the Pythagorean relationship

\[ a^2 + b^2 = c^2 \]
Explore the Pythagorean Relationship Using *The Geometer's Sketchpad®*

**Materials**
- computers
- *The Geometer's Sketchpad®* software

**Alternatives:**
- TECH 2.1A Explore the Pythagorean Relationship (GSP 4)
- TECH 2.1B Explore the Pythagorean Relationship (GSP 3)

**Technology Tip**
- Before you select a new object, make sure that you deselect first by clicking somewhere in the white space.

1. Open *The Geometer's Sketchpad®* and begin a new sketch.

2. Construct a right triangle.
   a) Use the **Straightedge** tool to create a horizontal line segment. Select the line segment and one endpoint. From the **Construct** menu, choose **Perpendicular Line**. Select the new line. From the Construct menu choose **Point on Perpendicular Line**. If the point is below the vertical line, move it up above. Select the perpendicular vertical line.

   ![Image of constructing a right triangle](image)

   b) From the **Display** menu, choose **Hide Perpendicular Line**. Select the three points. From the **Construct** menu, choose **Segments**.

3. Use the **Selection Arrow** to select all three line segments and all three points. From the **Display** menu, choose **Show Label**. Use the text tool to change the labels to read *a*, *b*, and *c*, as shown.

![Image showing labeled triangle](image)